Exercise 4

Compute $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$, where $\mathbf{a}$ and $\mathbf{b}$ are as in Exercise 3 and $\mathbf{c} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

Solution

The vectors are

$$\begin{aligned}\mathbf{a} &= (1, -2, 1)\\ \mathbf{b} &= (2, 1, 1)\\ \mathbf{c} &= (3, -1, 2).\end{aligned}$$

Now calculate $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$.

$$\begin{aligned}\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) &= (1, -2, 1) \cdot \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 2 & 1 & 1 \\ 3 & -1 & 2 \end{vmatrix} \\ &= (1, -2, 1) \cdot \left\{ \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} \hat{\mathbf{x}} - \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} \hat{\mathbf{y}} + \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} \hat{\mathbf{z}} \right\} \\ &= (1, -2, 1) \cdot \{[(1)(2) - (1)(-1)]\hat{\mathbf{x}} - [(2)(2) - (1)(3)]\hat{\mathbf{y}} + [(2)(-1) - (1)(3)]\hat{\mathbf{z}}\} \\ &= (1, -2, 1) \cdot (3\hat{\mathbf{x}} - \mathbf{y} - 5\hat{\mathbf{z}}) \\ &= (1, -2, 1) \cdot (3, -1, -5) \\ &= (1)(3) + (-2)(-1) + (1)(-5) \\ &= 3 + 2 - 5 \\ &= 0\end{aligned}$$